M (FY2011)

# Instruction: Mechano-Informatics (Subject)

Date : H22 (2010), August 24, 9:00 – 11:30

Answers should be written either in Japanese or English.

1) Do not open this problem booklet until the start of the examination is announced.
2) Among four problems provided, choose and answer two.
3) If you find missing, misplaced, and/or unclearly printed pages in the problem booklet, notify the examiner.
4) Two answer sheets are provided. Check the number of them, and if you find excess or deficiency, notify the examiner. You must use a separate sheet for each problem. When you run short of space to write your answer on the front side of the answer sheet, you may use the back side by clearly stating so in the front side.
5) In the designated blanks at the top of each answer sheet, write examination name "Mechano-Informatics (Subject)", "Master" or "Doctor" , your applicant number, and the problem number. Omission in filling up these blanks may void your test score.
6) An answer sheet is regarded as invalid if you write marks and/or symbols unrelated to the answer.
7) Submit both answer sheets even if they are blank.
8) Use the blank pages in the problem booklet for your draft.
9) Fill in your applicant number on the blank below, and submit this booklet. Also submit the Japanese booklet with your applicant number in the corresponding blank.

| Applicant number: |
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## Problem 1

P. 1. Answer the following questions about a two-link arm shown in Fig. 1. Torques and angles of two joints are $\boldsymbol{\tau} = [\tau_1 \quad \tau_2]^T$ and $\boldsymbol{q} = [\theta_1 \quad \theta_2]^T$, respectively. The link lengths are $l_1$ and $l_2$. A mass point of $m$ in mass is fixed at the tip P of the arm. The acceleration of gravity is $g$ in $x$ direction. Masses of links and joints, friction of joints, and the link deformations are ignored.

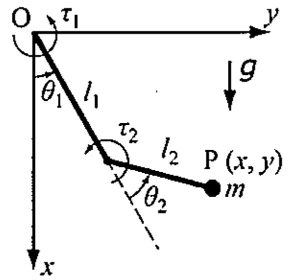


**Fig. 1**

(1) Obtain the Jacobian matrix $\boldsymbol{J}(\boldsymbol{q})$ which relates the velocity of the tip P $\boldsymbol{v} = [\dot{x} \quad \dot{y}]^T$ to the angular velocities $\dot{\boldsymbol{q}} = [\dot{\theta}_1 \quad \dot{\theta}_2]^T$.

(2) Suppose the situation of keeping static equilibrium by torques $\boldsymbol{\tau}$ against gravity. Under the condition that the measure of manipulability $w = |\det \boldsymbol{J}(\boldsymbol{q})|$ obtains the maximum value, calculate the joint angles $\boldsymbol{q}$ when the sum of the squares of the torques $\tau_1^2 + \tau_2^2$ becomes the minimum value. Assume that the link lengths are $l_1 = 1$, $l_2 = 1/\sqrt{2}$ and movable ranges of the joints are $-\pi/2 \le \theta_1 \le \pi/2$, $0 \le \theta_2 \le \pi$. Here, the sum of the squares of torques corresponds to energy consumption for keeping the joint angles constant.

P. 2. Let us measure a deflection and stretch of a beam when a mass point is fixed at the tip P as shown in Fig. 2. The beam is a uniform cantilever which is neither parallel nor perpendicular to the acceleration of gravity $g$. The cross section of the cantilever is rectangular. Strain gauges **a** and **b** are attached on the upper and lower sides of the cantilever, respectively. Consider the Wheatstone bridge consisting of the strain gauges **a** and **b** as well as two fixed resisters as shown in Fig. 3. When the strain gauge **a** is connected in the place of $r_1$, show the position of the strain gauge **b** among $r_2$, $r_3$ and $r_4$ with reasons, by separating such cases as measuring the deflection and measuring the stretch.

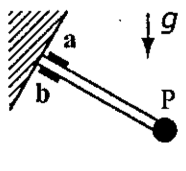


**Fig. 2**

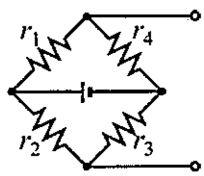


**Fig. 3**

P. 3. Figure 4 shows a circuit diagram of an instrumentation amplifier which amplifies the difference between $V_+$ and $V_-$ and outputs $V_{OUT}$. Answer the following questions. Use only following variables in the answers: $V_+$, $V_-$, $R_0$, $R_1$, $R_2$, and $R_3$. Symbols op1, op2 and op3 in Fig. 4 represent operational amplifiers.

(1) Considering the virtual short of the input terminals of the operational amplifiers, calculate the current $i$ that flows through the resister $R_0$.

(2) Considering that there is no current into the input terminals of the operational amplifiers, calculate the voltages $V_A$ at the point A and $V_B$ at the point B.

(3) Calculate the output voltage $V_{OUT}$.

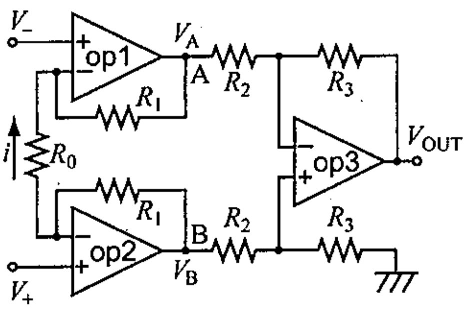


**Fig. 4**

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## Problem 2

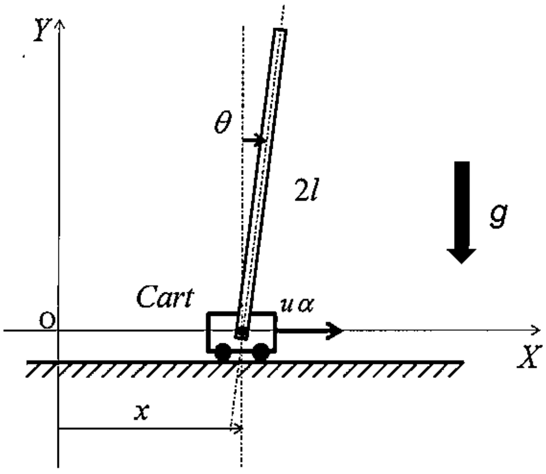


Fig. 1

As shown in Fig. 1, a uniform inverted pendulum with mass $M_2$ is attached to a cart with mass $M_1$ which moves on a rail along $X$ axis. The cart is moved by a motor which is connected to the cart's wheel. The position of the cart is defined as $x$. Let us design a system to control the movement of the cart and the angle of pendulum $\theta$ in $X$-$Y$ plane. When the input current $u$ is applied to the motor, the cart is driven by the force $u\alpha$ along with $X$ axis. The length of the pendulum is $2l$ and the acceleration of gravity is in the direction of $-Y$ axis with amplitude $g$. Assume the friction between the pendulum and the cart, and the friction loss between the cart and the rail are negligible and masses except the cart and the pendulum are ignored. The MKSA unit system is used. Answer the following questions.

(1) Give methods for measuring the cart's position $x$ and the angle of the pendulum $\theta$, and describe the principle and characteristics of each method.

(2) Calculate the gravity center of the pendulum $[x_G \quad y_G]^T$. Also calculate the kinematic energy $K$ and the potential energy $U$ of the system consisting of the cart and the pendulum.

(3) Let the Lagrange function $L$ be $L = K - U$. Show the Lagrange equations and equations of motion for $x$ and $\theta$.

(4) Assume $\theta$ is small enough and that $\sin\theta \cong \theta, \cos\theta \cong 1$, also assume $\dot{\theta}, \dot{x}$ are small and terms higher than second order are negligible. Then the equations of motion of (3) are described as the equation-of-state model of $\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{b}u$ , $\mathbf{y} = \mathbf{C}\mathbf{x}$ ,

where $\mathbf{x} = [x \quad \theta \quad \dot{x} \quad \dot{\theta}]^T, \mathbf{y} = [\theta \quad \dot{\theta}]^T$. When $\mathbf{A} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & A_{32} & 0 & 0 \\ 0 & A_{42} & 0 & 0 \end{bmatrix}, \mathbf{b} = \begin{bmatrix} 0 \\ 0 \\ b_3 \\ b_4 \end{bmatrix}$ and

$M_1 = 2, M_2 = 1, l = 1/\sqrt{3}$, derive $A_{32}, A_{42}, b_3, b_4$ and $\mathbf{C}$.

(5) It is known that the system of the previous equation-of-state model is controllable and stabilizable by state-feedback control. Consider the output feedback control system as shown in Fig. 2, where the reference value to the motor is $v(t)$ and the gain $\mathbf{K} = [k_1 \quad k_2]$. Discuss stability of the system, and obtain the range of $k_1, k_2$ that do not yield unstable roots.

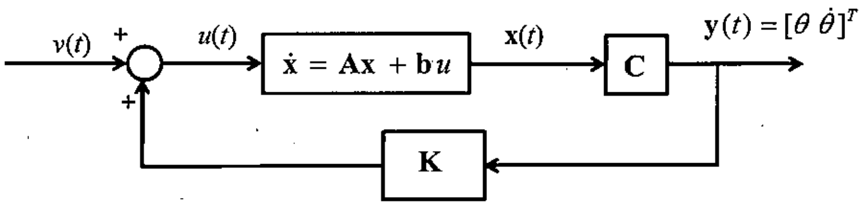


Fig. 2

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## Problem 3

P. 1. Describe the following terms in about three lines. You may use figures and tables if needed.

(1) Flip flop

(2) Parity

P. 2. Consider a linear function $f(\mathbf{x}) = w_0 + \mathbf{w}^T\mathbf{x}$ which models the relationship between input patterns and corresponding target values, where $\mathbf{w} = (w_1, w_2, \cdots, w_m)^T \in \mathrm{R}^m$ is a $m$ dimensional weight vector, and $w_0 \in \mathrm{R}$ is a bias. The training dataset consists of $N$ pairs of input patterns $\mathbf{x}_i = (x_{i1}, x_{i2}, \cdots, x_{im})^T \in \mathrm{R}^m$ $(i = 1, 2, \cdots, N)$ and target values $t_i \in \mathrm{R}$ $(i = 1, 2, \cdots, N)$. A sum-of-squares error between the input patterns and the target values in the training dataset is defined as $E = \sum_{i=1}^{N} (f(\mathbf{x}_i) - t_i)^2$. Answer the following questions.

(1) Suppose that $m = 2$ and $w_0$ is a constant. Find an expression $\mathbf{w} = (w_1, w_2)^T$ for the solution that minimizes $E$. Describe the conditions on the existence of solution if any.

(2) Suppose that $m$ is an arbitrary natural number. Find expressions $\mathbf{w} \in \mathrm{R}^m$ and $w_0 \in \mathrm{R}$ for the solution that minimizes $E$. Describe the conditions on the existence of solution if any.

P. 3. Consider a digital circuit of a function $h(\mathbf{x}) = H_c(w_0 + \mathbf{w}^T\mathbf{x})$ which outputs 1 or 0 binary value, where $\mathbf{x} = (x_1, x_2, x_3)^T$ is three dimensional binary patterns of 1s and 0s, $\mathbf{w} = (3, 2, 1)^T$, and $w_0 = -4$. $H_c(z)$ is a function whose value is 0 for $z < 0$ and 1 for $z \geq 0$. For example, if the input pattern is $\mathbf{x} = (1, 1, 1)^T$, the argument becomes $z = w_0 + \mathbf{w}^T\mathbf{x} = 2 \geq 0$. Therefore the value of $H_c(z)$ is 1.

Table 1

| $x_1$ | $x_2$ | $x_3$ | $h(\mathbf{x})$ |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | |
| 0 | 1 | 0 | |
| 0 | 1 | 1 | |
| 1 | 0 | 0 | |
| 1 | 0 | 1 | |
| 1 | 1 | 0 | |
| 1 | 1 | 1 | 1 |

(1) Complete Table 1 that shows a relationship between the input patterns $\mathbf{x} = (x_1, x_2, x_3)^T$ and the outputs of $h(\mathbf{x})$.

(2) Draw the digital circuit that represents the input-output relationship shown in Table 1 using as few gates as possible. Use only two-input AND gates, and two-input OR gates in the answer.

(3) Suppose that the input patterns, the weight, and the bias are modified to four dimensional binary patterns of 1s and 0s $\mathbf{x} = (x_1, x_2, x_3, x_4)^T$, a weight $\mathbf{w} = (4, 3, 2, 1)^T$, and a bias $w_0 = -5$ respectively in the function $h(\mathbf{x}) = H_c(w_0 + \mathbf{w}^T\mathbf{x})$. Draw the digital circuit that represents the input-output relationship of the modified function using as few gates as possible, and describe the process to derive the circuit. Use only two-input AND gates, and two-input OR gates in the answer.

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## Problem 4

P. 1. Describe the following terms in about three lines. You may use figures.

(1) Stack

(2) Functional language

P. 2. Consider addition, multiplication, and division of positive binary numbers in CPU. Answer the following questions.

(1) Express the sum of binary numbers 11111111 and 11111111 in binary representation. Express the sum in hexadecimal representation, too.

(2) The product of 4bit binaries 1010 and 1001 can be obtained as in Fig. 1. Express the product of binaries 111111 and 111111 in binary representation. Express the product in hexadecimal, too.

```
       1010
   ×   1001
   --------
       1010
      0000
     0000
    1010
   --------
   01011010
```

**Fig. 1**

(3) Express the quotient in binary representation when 1111101100000110 is divided by 11111110. Express the quotient in **decimal** representation. Describe the calculation process with the result.

(4) When a multiplier is m bit and a multiplicand is n bit, how many bits are required to store the product?

(5) Figure 2 shows a basic configuration of 2p bit (ex. 16bit) binary multiplication hardware. The multiplier is put into 2p bit multiplier register, the multiplicand is put into the lower 2p bit of 4p bit (ex. 32bit) multiplicand register, and the product register which keeps the product is initialized as 0 when calculation starts. Hereafter, the product is calculated as follows. Fulfill the five [ ] below.

*a.* Judges whether the lowest bit of the multiplier is 0 or 1.

*b.* If 1, adds the multiplicand to the product register value, and puts the result into the product register.

*c.* Shifts the multiplicand register by [ ① ] bit [ ② ]-hand side.

*d.* Shifts the multiplier register by [ ③ ] bit [ ④ ]-hand side.

*e.* Judges the repetition number of step *a* to step *d*, and if the number is less than [ ⑤ ] times, then goes to step *a*.

*f.* Finishes the multiplication.

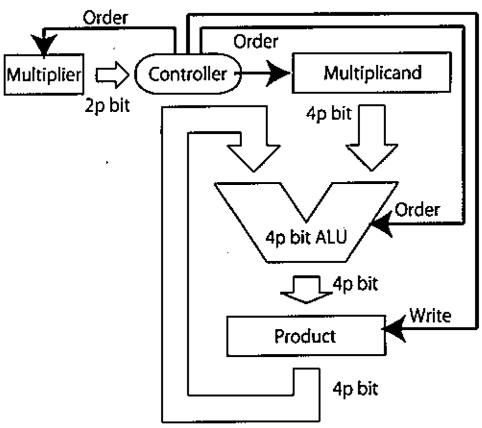


**Fig. 2**

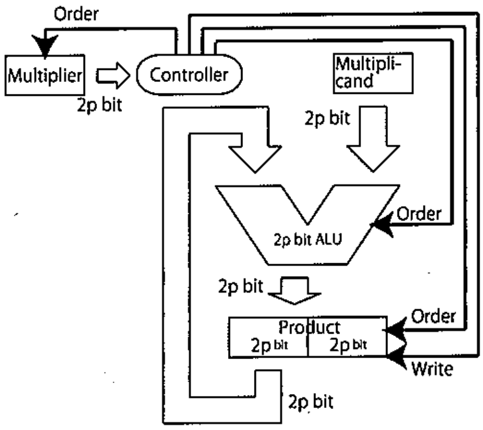


**Fig. 3**

(6) In 2p bit multiplication method of (5), the half of the multiplicand register's bits are always 0. And the half of bits added to the intermediate result are always 0. These are inefficient, so the method that uses 2p bit multiplicand register and 2p bit ALU is proposed as shown in Fig. 3. The step *b* and step *c* of the method of (5) need to be changed. Describe the process of each step.

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