2010 Winter Entrance Examination

Department of Creative Informatics
Graduate School of Information Science and Technology
The University of Tokyo

Programming

INSTRUCTIONS

1. Do not open this problem brochure until the signal to begin is given.
2. Write your examinee ID below on this cover.
3. An answer sheet and a draft sheet accompany this brochure. Write down your examinee ID on these sheets.
4. The USB memory delivered beforehand to each examinee contains ASCII text files: `a.txt`, `b.txt` and `c.txt`. Newline is represented by carriage return (CR) followed by line feed (LF) in the file.

   Before the examination starts, copy these files to your PC and browse them. Make sure you can see text files and keep your hands away from your PC. If you cannot read the files properly, consult the test supervisor. The contents of the USB memory are common to all examinees.
5. You may choose your favorite programming languages.
6. You may consult only one printed manual of a programming language in the examination. *You can use or copy any libraries or program segments existing in your PC, but you cannot connect to the Internet.*
7. By the end of the examination, make a directory/folder on your PC, whose name is the same as your examinee ID, and put your program files and related files into the directory/folder. Copy the directory/folder onto the delivered USB memory.
8. At the end of the examination, the USB memory, the answer sheet and the draft sheet are collected.
9. After these are collected, stay at your seat, until all examinee program results have been checked briefly by the test supervisor.
10. After the brief check, try to save your program execution environment on the PC so that you can run your program as soon as possible during the oral examination in the afternoon.
11. *Leave your PC and this brochure together in the room* for the oral examination and leave the room until you are called.

Examinee ID _____

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Consider a directed graph whose vertices and arcs (directed edges) increase and decrease with time.

Let $G(t) = \langle V(t), A(t) \rangle$ denote a directed graph at time t, where $V(t)$ is a set of vertices and $A(t)$ is a set of arcs at time t. Let (v_x, v_y) denote an arc from a vertex v_x to v_y. The initial state is $G(0) = \langle V(0), A(0) \rangle$ where $V(0) = \{v_0\}$ and $A(0) = \{\}$. Let $R(t)$ be a root-set: the set of vertices consisting of the vertex v_0 and all reachable vertices from v_0 at time t. Reachable here means the vertex can be reached after traversing any number of arcs.

Each line of text files in the USB memory describes an operation that increases or decreases vertices or an arc. The operation at line t finishes at time t.

Answer the following questions from Q1 to Q3. Write your answers to all questions on the answer sheet. Write your answers referring to the output of your computer program if there are no other instructions. You do not need to write down your program on the answer sheet. You are required to explain your program at the oral examination.

Q1 Consider a directed graph where vertices and arcs increase with time.

An operation **Add-VA** is defined as follows: given a vertex v_x followed by a vertex v_y, apply the following operations to $G(t-1)$ and generate $G(t)$[†1] :

$$V(t) = V(t-1) \cup \{v_x, v_y\} \quad (1)$$
$$A(t) = A(t-1) \cup \{(v_x, v_y)\} \quad (2)$$

In the text file `a.txt` and other text files, the operation **Add-VA** is denoted as a line:

$$x\text{->}y$$

where x and y are integers from 0 to 10000, which correspond to vertices from v_0 to v_{10000}, respectively. The operation at line t finishes at time t. An example is shown on the next page.

The directed graph $G_a = \langle V_a, A_a \rangle$ is the graph which results after applying all operations in the text file `a.txt` to $G(0)$. Answer the following questions.

1-1 Write the number of vertices $|V_a|$ of the graph G_a.

1-2 Write one of the vertices which have the maximum out-degree in the graph G_a and its out-degree. Similarly, write one of the vertices which have the maximum in-degree in the graph G_a and its in-degree[†2] .

1-3 Write the time t_v where $|V(t_v - 1)| < 1000$ and $|V(t_v)| \geq 1000$. Similarly, write the time t_r where $|R(t_r - 1)| < 1000$ and $|R(t_r)| \geq 1000$.

1-4 Write the time t when the vertex v_0 creates a cycle for the first time.

[†1] The meaning of equation (1) is that if the vertex v_x is not an element of $V(t-1)$, the operation adds v_x to $V(t-1)$ to generate $V(t)$. The same applies to v_y. The meaning of equation (2) is that if the arc (v_x, v_y) is not included in $A(t-1)$, the operation adds (v_x, v_y) to $A(t-1)$ and generate $A(t)$.

[†2] The out-degree of a vertex v is the number of arcs going out of v. The in-degree of a vertex v is the number of arcs coming into v.

Example of Q1 If a text file shown in Figure 1 is given, then Table 1 shows the set of vertices $V(t)$, the set of arcs $A(t)$ and the root-set $R(t)$ at each time.

Finally, the number of vertices $|V(5)|$ is 6, the size of the set of arcs $|A(5)|$ is 5, the size of the root-set $|R(5)|$ is 5. Figure 2 also depicts the directed graphs from time 0 to 5.

```
0->1
2->3
3->4
3->5
1->3
```

Figure 1: An input example of Q1

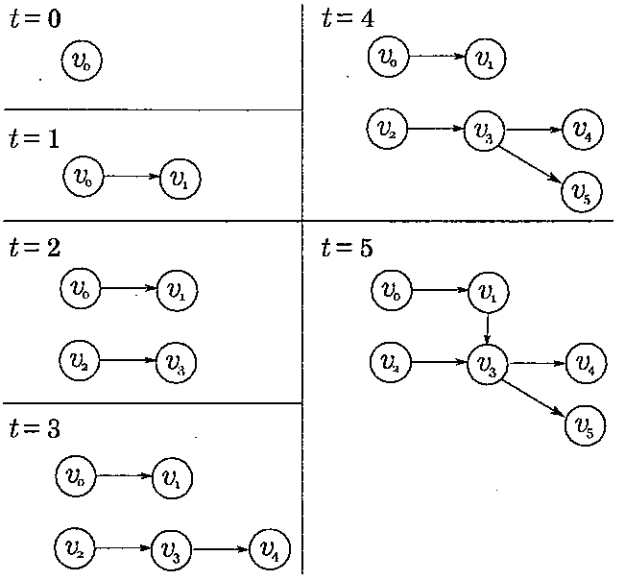

Figure 2: Directed graphs at each time for the example of Q1

Table 1: The consequence of an input for the example of Q1

t	$V(t)$	$A(t)$	$R(t)$
0	$\{v_0\}$	$\{\}$	$\{v_0\}$
1	$\{v_0, v_1\}$	$\{(v_0, v_1)\}$	$\{v_0, v_1\}$
2	$\{v_0, v_1, v_2, v_3\}$	$\{(v_0, v_1), (v_2, v_3)\}$	$\{v_0, v_1\}$
3	$\{v_0, v_1, v_2, v_3, v_4\}$	$\{(v_0, v_1), (v_2, v_3), (v_3, v_4)\}$	$\{v_0, v_1\}$
4	$\{v_0, v_1, v_2, v_3, v_4, v_5\}$	$\{(v_0, v_1), (v_2, v_3), (v_3, v_4), (v_3, v_5)\}$	$\{v_0, v_1\}$
5	$\{v_0, v_1, v_2, v_3, v_4, v_5\}$	$\{(v_0, v_1), (v_2, v_3), (v_3, v_4), (v_3, v_5), (v_1, v_3)\}$	$\{v_0, v_1, v_3, v_4, v_5\}$

Q2 Consider the case of the introdution of the deletion of arcs.

In addition to the operation **Add-VA**, the operation **Del-A** is defined as follows: given an arc (v_x, v_y), the following operation is applied to $G(t-1)$ to generate $G(t)$[†3] :

$$A(t) = A(t-1) \setminus \{(v_x, v_y)\} \tag{3}$$

Similar to the operation **Add-VA**, the operation **Del-A** is denoted as follows:

$$!x\text{->}y$$

The directed graph $G_b = \langle V_b, A_b \rangle$ is the graph which results after applying all operations in the text file `b.txt` to $G(0)$. Answer the following questions.

2-1 Write the number of arcs $|A_b|$ of the graph G_b.

2-2 Write the size of the root-set $|R_b|$ of the graph G_b.

2-3 Write all the times t where $|R(t-1)| < 1000$, $|R(t)| \geq 1000$.

Q3 Consider the case that vertices are deleted with arc deletions.

Execute the following operation **S1** after the operation **Add-VA** or **Del-A** every time.

> **S1** Delete every vertex that is not an element of the root-set $R(t)$. Also delete arcs going out of the deleted vertex v and arcs coming into v.

The directed graph $G_c = \langle V_c, A_c \rangle$ is the graph which results after applying all operations in the text file `c.txt` with the operation **S1** to $G(0)$. Answer the following questions.

3-1 Write the number of vertices $|V_c|$ and the number of arcs $|A_c|$ of the graph G_c.

Instead of the operation **S1**, execute the following operation **S2** after the operation **Add-VA** or **Del-A** every time. The directed graph $G'_c = \langle V'_c, A'_c \rangle$ is the graph which results after applying all operations in the text file `c.txt` with the operation **S2** to $G(0)$.

> **S2** Delete every vertex v except v_0 where the in-degree of v is 0. Also delete all arcs going out of v. If this operation generates another vertex v where the in-degree of v is 0, apply the operation **S2** to the resulting graph again.

3-2 Write the number of vertices $|V'_c|$ of the graph G'_c.

3-3 Analyze the difference between the number of deleted vertices with the operation **S1** and the operation **S2**.

[†3] The meaning of the equation (3) is that if $A(t-1)$ contains the arc (v_x, v_y), then the operation removes (v_x, v_y) from $A(t-1)$ and generates $A(t)$. $A \setminus B$ means the set of all elements which are members of A but not of B.

Example of Q2 If a text file shown in Figure 3 is given, then Table 2 shows the set of vertices $V(t)$, the set of arcs $A(t)$ and the root-set $R(t)$ at time t.

```
0->1
2->3
1->2
!2->3
1->3
```

Figure 3: An input example of Q2

Table 2: The consequence of an input for the example of Q2

t	$V(t)$	$A(t)$	$R(t)$
0	$\{v_0\}$	$\{\}$	$\{v_0\}$
1	$\{v_0, v_1\}$	$\{(v_0, v_1)\}$	$\{v_0, v_1\}$
2	$\{v_0, v_1, v_2, v_3\}$	$\{(v_0, v_1), (v_2, v_3)\}$	$\{v_0, v_1\}$
3	$\{v_0, v_1, v_2, v_3\}$	$\{(v_0, v_1), (v_2, v_3), (v_1, v_2)\}$	$\{v_0, v_1, v_2, v_3\}$
4	$\{v_0, v_1, v_2, v_3\}$	$\{(v_0, v_1), (v_1, v_2)\}$	$\{v_0, v_1, v_2\}$
5	$\{v_0, v_1, v_2, v_3\}$	$\{(v_0, v_1), (v_1, v_2), (v_1, v_3)\}$	$\{v_0, v_1, v_2, v_3\}$

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